The Impact of a System of Intelligent Agents on a Market at Equilibrium

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EXTENDED ABSTRACT

Bandwidth gains achieved in telecommunication technology have been followed by a recent proliferation of Intelligent Agents on the internet. Intelligent agents have emerged as mechanisms that aid both buyers and sellers in very different ways. It has been observed by several researchers that electronic search mechanisms promote "frictionless" competitive markets that lower buyers' search costs and seller profits (e.g., see Bakos 1987, 1991, 1997; Malone, Benjamin and Yates 1991). While some research has been done on buyer search for sellers, there are very few formal models of seller search for buyers. Intelligent agents (IAs) facilitate seller search for buyers. IA's also work for buyers and reduce the aggregate disutility of a purchase decision (price, search and comparison related costs). These IA's are called Decision Agents [see Do, O. et al]. Demand agents on the other hand, work for sellers and perform two important functions; first they infer buyer preferences and estimate buyer reservation prices and second, they supply decision agents with product information. Demand agents infer buyer preferences based on the characteristics of the buyers' context such as site demographics, buyer demographics and by employing interactive information collation techniques such as collaborative filtering, observed buyer behavior modification and analysis of self reported data [see Do, O. et al].

IAs work in two different modes; Centralized and Distributed (or single agent and multi-agent systems) [multi agent]. A centralized IA analyzes buyer profiles using a variety of techniques (such as the use of "cookie files"). A system of distributed agents (multi-agent system) is required when it is necessary to analyze the interactions and behavior patterns of different people or organizations with different (possibly conflicting) goals and context dependent proprietary information [multi agent]. Distributed agent systems construct a buyer's profile by observing the buyer's behavior on the internet and by creating a history of past purchase behavior along with details of sites visited and buyers' interests. The buyer profiles are shared by the agents which anticipate user needs and make offers to them. The best example of such agents is IBM's Memory Agent, which works as a distributed demand agent [IBM Agent]. IAs such as Personal View, Firefly, NewsHound, Fishwrap etc. are examples of centralized agents while IBM's Memory Agent, Similarity Engine, NewsWeeder, BotShop etc. are examples of distributed agents.

In this paper we model the impact of IAs on a market for a highly differentiated product. Further, we consider how the two different agent architectures impact sellers' pricing strategies and buyers' buying decisions. We examine the extent to which IA's support price discrimination on the part of sellers. We also examine if there is a significant difference in the market outcomes that are brought about by the two kinds of IAs and if there exist incentives for investment in technology.

The primary contribution of this research is two-fold. First, by modeling the functioning of IAs at a more detailed level than previous studies, we provide a finer understanding of how IAs impact buyer market outcomes, and the resulting implications for buyer and seller surplus. Second, this research provides guidelines for the design of IA's that will best serve different kinds of markets and maximize seller profits, buyer surplus, or total welfare. Our approach is to model through closed form equations the impact of IAs on a market, and the resulting firm profits as characterized by the IA's precision, cost of technology and the choice of platform (centralized / distributed). Where closed form solutions are infeasible, we will provide simulations of interesting scenarios.

Outline of Model

We construct two different models one each to model the functioning of centralized and distributed IAs. We first discuss the features that are common to both models and then delineate the features that are specific to a model when we address the impact of a particular type of architecture.

Buyers and Sellers

1. In a market consisting of many sellers, one seller introduces an IA. Due to an initial asymmetric technological endowment, other sellers lack the capacity to introduce the agent[1]. This seller will be referred to as "the seller" for the remainder of this paper for expositional simplicity.
2. There are *N* Buyers in the market who have heterogeneous reserve utilities uniformly distributed between [U_{min} , U_{max}] which is normalized to [0,1] without loss of generality.
3. The Seller who introduces an IA offers highly differentiated information rich products whose marginal cost is set as C_0 for a range of quantities produced and sold. C_0 is treated as a very small fraction of U_{min}.[2]

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[1] The reasons for the asymmetry in technological endowment are exogenous to the model.

[2] This is in consonance with actual marginal cost schedules of several information rich products such as music, electronic versions of books, entertainment, information, stock quotes etc. where the cost of producing an incremental unit is low enough to be treated as a constant and insignificant in comparison to the reserve utility of the buyer.

4. The seller's offer to a buyer is made through an IA and is therefore not known to any other buyer.
5. The Seller reaches buyers directly using an IA and shuts other sellers out of the market by eliminating the need for buyer search for products. This allows the seller to behave as a monopoly. The seller's monopoly price offering to the i^{th} buyer is simply the reserve utility U_i of the i^{th} buyer (note that the seller does not know U_i and has only an estimate of it).
6. The seller make a single offer to buyers using an IA and does not indulge in multi-period bargaining scenarios. A buyer will accept the offer if and only if, it is priced below his reserve utility.

Centralized Intelligent Agent Model

Intelligent Agent

1. An agent infers a buyer's reserve utility and estimates it within an interval. ***The Error Factor*** (a proxy of inaccuracy) of the IA's estimate is given by , where, *0 ≤ ≤ 1*. This means that when an agent estimates the i^{th} buyer's reserve utility to be to U_i, at an error factor (percentage) of , the real reserve utility of the i^{th} buyer given by U_i is such that:

$$U_i \in \left[U'_i(1-q), U'_i(1+q)\right]$$

   Further, the real reserve utility U_i is ***uniformly distributed***[3] in [*U i (1-) , U i (1+)*].

2. The seller will make an offer priced at P_i to the i^{th} buyer such that his total expected revenue will be maximized. In Figure1 below, the IA has estimated a buyer's utility to be *U*. This implies that the buyer's real reserve utility, say, U_0 is such that:

$$U_0 \in [U(1-q), U(1+q)] \text{ and } U_0 \text{ is uniformly distributed on } [U(1-q), U(1+q)] \text{ such that}$$

$$\text{F(u)} = \frac{\text{u - }U(1-q)}{2Uq} \text{ and } f(u) = \frac{1}{2Uq}$$

3. In Figure 1 (overleaf),if the seller makes an offer to the buyer priced at *x*, the buyer will accept the offer if his reserve utility is to the right of *x* and will reject it if it is to the left of *x*. A central issue for the seller is to identify the profit maximizing price as a function of *U* and .

4. The cost of technology is given by *C* which increases as the IA's error factor decreases. We have modeled it as a quadratic function of and have used the market size as a scaling factor. Cost *C* is modeled as:

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[3] While the actual value of U_i is a constant, the seller does not know it and can only treat it as a random variable and attach different probabilities to different possible values of U_i. If a buyer's reserve utility is estimated as 0.8 by an agent which has an error factor of 25%, then the real reserve utility of that buyer can lie anywhere between [0.6 to 1].

$$C(q,N)=\frac{q^{-2}+q^{-1}}{Ln(N)}$$

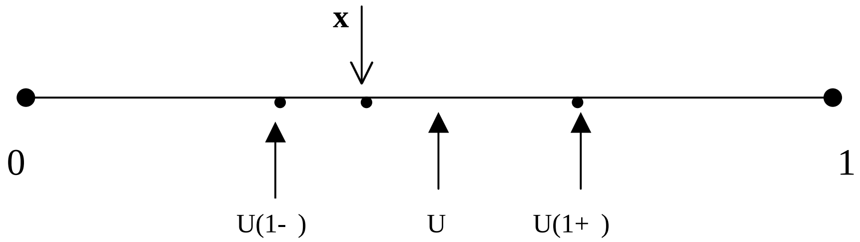

Figure 1: Buyer's reserve utility and seller's offer

Expected Revenue from making a price offering of $\mathrm{r}=\frac{(U(1+q)-x)x}{2Uq}$

$$\frac{dr}{dx}=\frac{d(\frac{[U(1+q)x-x^2]}{2Uq})}{dx}=\frac{[U(1+q)-2x]}{2Uq}$$

To determine the maximum expected revenue we take first order conditions which results in : $\frac{dr}{dx}=0$

$$\Rightarrow\frac{[U(1+q)-2x]}{2Uq}=0\Rightarrow x=\frac{U(1+q)}{2}$$

This results in the best price only if $\frac{U(1+q)}{2}\leq U(1-q)$, or if $q<\frac{1}{3}$.

Similarly, by checking for different boundary conditions, we derive a set of equations that link the best price P* with estimated buyer reserve utility U and error facto$q$, shown below.

Table 1: - Best offered Price, P*, for different values of U and θ

	Location of the Estimated Reserve Utility *U*		
Error factor -	**U < 1/(1+θ)**	**1/(1+θ)<= U <= 1/2(1-θ)**	**U > 1/2(1-θ)**
q<= 1/3	Best Price is Always: P*=U(1-θ)		
1/3 <q <=1/2	P*=U(1+θ)/2	P*=1/2	P*=U(1-θ)
q>1/2	P*=U(1+θ)/2	P*=1/2	

Each value of revenue maximizing price P^*, above, generates a corresponding value of revenue which is a function the number of buyers N in the market and the error factor .
A brief sketch of the derivation of the expression for expected revenue is provided below for the Case 1/3 ≤ ≤1/2 and 1/(1+)≤U≤1/2(1-).

Let R be the revenue that is generated by a single buyer whose reserve utility is estimated as U.

$$R = P^* \times \Pr(\text{the buyer will buy the product}) = \frac{(1/2)(1-1/2)}{[1-U(1-q)]} = \frac{1}{4(1-U(1-q)]}$$

Since U is uniformly distributed in $[\frac{1}{(1+q)}, \frac{1}{2(1\text{-}q)}]$, R is Uniformly distributed in $[\frac{1+q}{8q}, \frac{1}{2}]$

$$F(r) = P(R \le r) \Rightarrow F(r) = \frac{r - \frac{1+q}{8q}}{(\frac{1}{2} - \frac{1+q}{8q})} = \frac{8rq - 1 - q}{3q - 1} \Rightarrow f(r) = \frac{8q}{3q - 1}$$

$$E(R) = \int_{\frac{1}{(1+q)}}^{\frac{1}{2(1-q)}} r f(r)dr = \left(\frac{8q}{3q-1}\right)\int_{\frac{1}{(1+q)}}^{\frac{1}{2(1-q)}} r dr = \frac{1+5q}{16q}$$

Since the number of buyers in the interval can vary from 0 to N, the expected revenue from all the buyers in that interval is[4]:

$$\sum_{i=0}^{N}\left[i\left(\frac{1+5q}{16q}\right)\left(\frac{1-3q}{2(1-q^2)}\right)^{i}\left(1-\frac{1-3q}{2(1-q^2)}\right)^{N-i}\binom{N}{i}\right]$$

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[4] Proof of this result will be made available by the authors on request.

The following table gives the Expected Total Revenue, $R(N,q)$, for various values of q :

$Case1: q < \frac{1}{3}$; Expected Total Revenue, $R(N,q) = \frac{N(1-q)}{2}$

$Case2: \frac{1}{3} \leq q \leq \frac{1}{2}$; Expected Total Revenue, $R(N,q) =$

$$\sum_{i=0}^{N}\sum_{j=0}^{N-i}\left[\left(\frac{1}{1+q}\right)^{i}\left(\frac{3q-1}{2(1-q^{2})}\right)^{j}\left(\frac{1-2q}{2(1-q)}\right)^{N-(i+j)}\binom{N}{i}\binom{N-i}{j}\left(i\frac{(3-2q)}{4}+j\frac{1+5q}{16q}+(N-i-j)\frac{3-2q}{4}\right)\right]$$

$Case3: q > \frac{1}{2}$; Expected Total Revenue, $R(N,q) =$

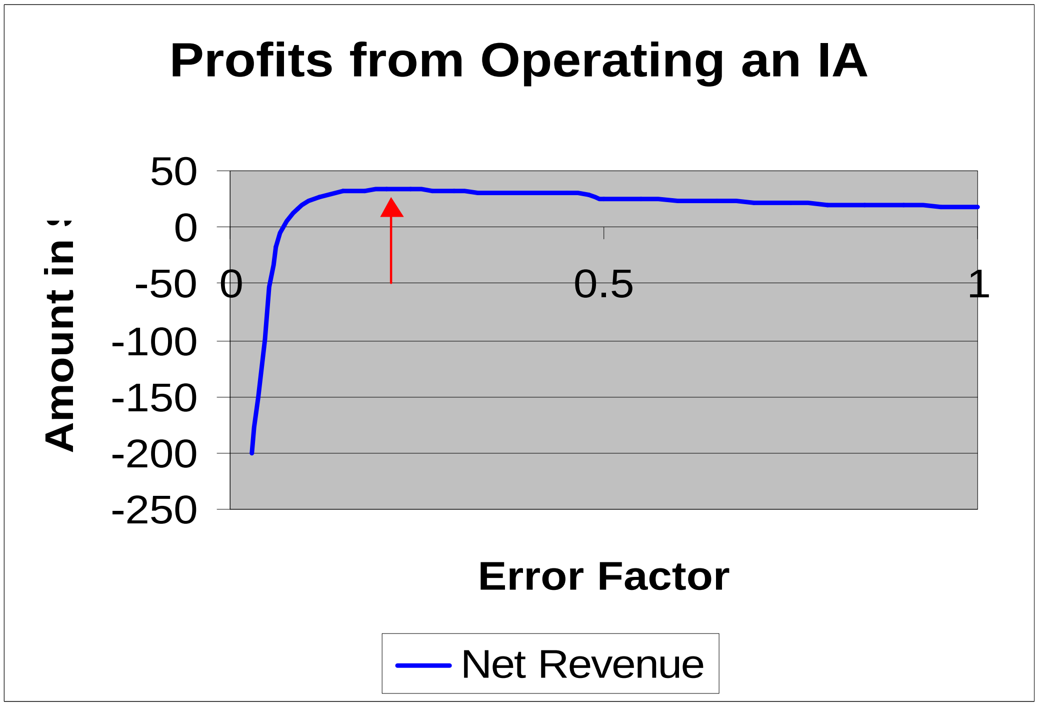
$$\sum_{i=0}^{N}\left[\left(\frac{1}{1+q}\right)^{i}\left(\frac{q}{1+q}\right)^{N-i}\binom{N}{i}\left(i\left(\frac{1+q}{16q}\right)+(N-i)\left(\frac{3+q}{16q}\right)\right)\right]$$

Profits generated by operating the IA :

$$\prod_{(N,q)} = R(N,q) - C(N,q)$$

We will solve the above equation to determine the optimal precision (error factor) and the resulting firm profits. Preliminary results indicate that minimizing the error factor does not automatically maximize profits and that there is an optimal vale that maximizes profits.

Figure 2: Optimal profits and Error Factor (arrow points to $_{\max}$).



Distributed Intelligent Agent Model

In this section we will expand our model in two important aspects. First we will add the dimension of quality and uncertainty arising from product quality and secondly, we will deal with products that are imperfect substitutes.

1. We assume that a monopoly firm[5] is producing two goods that are imperfect substitutes. The quantity of good j is given by x_j.
2. There are two IAs A and B.
3. There is a continuum of consumers in the interval $[0,1]$ indexed by h whose utility function is quadratic in the goods produced and additively separable with respect to money. The representative consumer's utility in the differentiated goods is given by

$$u(x) = [a + (b - g)q]x_1 + [a - (b - g)q]x_2 - \frac{1}{2}[bx_1^2 + 2gx_1x_2 + bx_2^2]$$

$$a, b, g > 0$$

$$b \geq g$$

4. If $g = 0$, products are independent and for $g = b$, they are perfect substitutes. Consumers have limited information about the parameter q distributed $N\left(m, \frac{1}{R}\right)$ which represents the matching between product characteristics and consumer preferences. As the value of q increases, the utility derived from consuming good 1 increases while that of 2 decreases. The demand agents (DA) have a precision level[6] R which decreases the variance of q as R increases. The relation between q and R is given below. Here, µ is the actual quality difference of the products and the precision of the agent decreases the variance around this value by providing a better match to consumers' preferences. The expected quality, given the precision level is R, is given by $\bar{q}(R) = \int_{-\infty}^{+\infty} qf(q)dq$.

From expected utility maximization and price taking behavior, we can write the demand functions as follows:

$$x_1 = \frac{a}{b+g} + \bar{q}(R) - \frac{b}{b^2 - g^2}p_1 + \frac{g}{b^2 - g^2}p_2$$

$$x_2 = \frac{a}{b+g} - \bar{q}(R) - \frac{b}{b^2 - g^2}p_2 + \frac{g}{b^2 - g^2}p_1$$

The decentralized equilibrium will lead each agent to maximize its own profits:

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[5] A firm that can introduce an IA into a market when its competitors cannot can adopt monopolistic pricing under conditions discussed earlier.

[6] This is not the same as the error factor , from the previous model. R is a direct measure of the ability of the IA to diminish the uncertainty surrounding the extent to which a product matches a buyer's preferences.

$$\Pi_A = \left(\left(\frac{a}{b+g} \right) + \bar{q}(R_A) - \frac{b}{b^2 - g^2} p_1 + \frac{g}{b^2 - g^2} p_2 \right) p_1 - cR_A$$

$$\Pi_B = \left(\left(\frac{a}{b+g} \right) - \bar{q}(R_B) - \frac{b}{b^2 - g^2} p_2 + \frac{g}{b^2 - g^2} p_1 \right) p_2 - cR_B$$

5. We will assume that the each agents will make offers given the other agent’s price offer. Hence although the seller acts as if he is a monopoly producer, since we are studying the decentralized case, the agents are assumed to act like oligopolistic firms (themselves) whose profits accrue to the seller.

6. Equilibrium prices are given as follows:

$$p_1^* = \frac{1}{4} \frac{(b-g)(2bg + 4b^2 - g^2)(\bar{q}(R_A)(b+g) + a)}{(2b^2 - g^2)b}$$

$$p_2^* = \frac{1}{2} \frac{(2b+g)(b-g)(\bar{q}(R_B)(b+g) + a)}{2b^2 - g^2}$$

In equilibrium, $R_A > R_B \Rightarrow \Pi_A > \Pi_B$ condition would lead to the intuitive result that the higher precision agent would reap higher profits. In general, we will show that, if higher precision implies higher level of match, $R_A > R_B \Rightarrow q(R_A) > q(R_B)$, then the marginal cost of additional precision has to be small enough to justify the existence of a high quality precision agent together with a low precision one.

Research Plan

1. We will examine if there incentives for investment in technology and if it is viable for a third party to invest in and operate a system of IAs that could be used by sellers.
2. We will examine if the nature of product and the extent of uncertainty about its quality, drives the choice of IA platform.
3. We will investigate the extent of price discrimination that sellers can practice by introducing an IA in the market.
4. We will provide simulations to examine the relative impact of market size, IA precision, cost of technology and other parameters on seller profits and price dispersion.
5. We will explore the extension of this model to include oligopolistic competition between sellers who operate IAs.

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